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Stability analysis of a squealing vibration model with time delay

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Abstract

An exact stability analysis of a squealing vibration model with time delay is carried out using Olgac's direct method. The stability regions of the model without squealing noise are identified. Sensitivity of parameters, including a natural frequency, damping and an integrated coefficient of contact and friction, to the occurrence of squealing vibration is analysed. The results show that the time delay between the varying normal force and its causing varying friction has a distinct influence on the occurrence of squeal. It is found that with an increase of the time delay, the stability regions without squeal and the unstable regions with squeal generally arise alternately. The possibility of squeal occurrence is found to increase with increasing natural frequency. It is also found that the larger the integrated coefficient, the more easily squealing vibration occurs. The nonlinear simulation result shows that the contact separation between the two sliding surfaces is a main nonlinear factor which leads to squealing vibration being bounded. Disappearance of squeal is predicted. The simulation results are compared with the test results in both time and frequency domains and is found to be in good agreement with test results. The mechanism of squeal generation, growth and disappearance is proposed.

1. Introduction

Automobile brake noise and vibration have become an increasing concern to manufacturers, consumers and investigators. Friction can result in a series of acoustic problems [1]. Brake squeal may be considered to be a type of friction-induced noise. Friction-induced noise is generally divided into two categories: squeal or squeak and chatter, groan, or moan [1–3]. No precision definition of brake squeal has gained complete acceptance, but it is generally accepted that squeal is a sustained, high frequency (>1000 Hz) vibration of brake system components resulting in noise audible to vehicle occupants or passers-by [2]. Chatter, groan or moan is mainly used to term a category of noise whose frequency is below 1000 Hz. The sound pressure level of squeal generally falls into 65–90 dB and the maximum may reach 100–120 dB.

Brake squeal is an annoying noise induced by friction. Its generation and growth strongly depend on interaction in contact and friction between two sliding surfaces and vibration mode characteristics of the friction system. Brake squeal deals with interdisciplinary knowledge including tribology, acoustics, contact and vibration mechanics [1–6]. The investigation into friction-induced noise may be traced back to the 1930s. Up to now, there are a rich number of publications on the subject in the literature. Several comprehensive

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reviews were published [1–6]. Six mechanisms of squeal formation were summarized in these reviews. They are stick-slip, negative friction–velocity slope, sprag-slip, modal coupling, splitting the doublet modes and hammering. These mechanisms are indispensable for better understanding of squeal. Recently, a finite element complex eigenvalue analysis has become a popular tool for squeal prediction [7–11]. That approach is generally considered to be capable of predicting squeal onset and of suppressing squeal in the design stage of products. Since a finite element complex eigenvalue analysis is based on a linear model, it can be applied only to study the squeal onset. It cannot be applied to study the growth and disappearance of squeal because there is some nonlinearity in these stages. Most recently, several finite element transient analyses of squeal, were reported, which deal with some nonlinearity [12,13]. These transient analyses have been successfully used to simulate the onset process and limit cycle vibration of squeal. In discrete modelling of squealing brake systems, over 15 different models were reported in the literature [1–3,14–17]. These models involve 2–14 degrees of freedom. They have led to understanding the squeal generation mechanism well, through modelling these friction systems.

One may find all six mechanisms mentioned above can be applied only to explain the initiation of squeal. That is the main limitation of these mechanisms. However, what limits infinite increase of squealing vibration is seldom studied and so also is the question what results in disappearance of squealing vibration. Since friction between two sliding surfaces is very complicated and some friction feature characteristics probably remain unknown, the current understanding of squeal has not led to a satisfactory dilution closure to the squeal problem. As shown in the Ouyang's work [18], using the velocity-dependent friction the region of instability was bounded in a finite speed range. Otherwise, if the constant friction was used, the region of instability for the same parameters does not have an upper bound. Kinkaid commented that 'the development of a comprehensive predictive model of disc brake squeal is the ultimate goal of most of the investigators. This goal has been elusive, and whether or not such a model is possible remains to be seen' [2]. That suggests that there is still much work needed to be done for the goal.

The present authors applied the time–frequency analysis technique to successfully extract a nonlinear characteristic of vibration-associated squeal [19]. It was reported that for squealing vibration there was always a decrease in frequency at the location where the vibration is bounded. A new concept of the time delay between the varying normal force and its causing changing friction was introduced and verified partly [20]. Based on the time delay concept, a self-excited vibration model with time delay was proposed to explain the formation mechanism of squealing noise [20]. That work brings a new insight on the squeal generation mechanism. But a detailed theory analysis of the squealing vibration model with time delay was not presented. In fact, a vibration model with time delay exhibits very complicated vibration behaviour [21,22]. A comprehensive understanding of the complicated vibration behaviour is profitable for understanding the mechanism behind squeal.

The purpose of the paper is to present a detailed theory analysis of stable and unstable regions of the squealing vibration model with time delay. Based on the stability analysis result, linear and nonlinear simulations of the model are carried out. The simulation results are compared with test results in both time and frequency domains. A prediction of squeal disappearance is performed using the nonlinear model with time delay. Moreover, sensitivity of parameters to squeal propensity is also analysed.

2. Self-excited vibration model and its stability analysis approach

2.1. Brief introduction to the self-excited vibration model with time delay

The present authors proposed a self-excited vibration model [20]. It is mainly composed of an energy feedback loop and a time delay term as an exciting source. Whether one is using a sliding friction system or a pin on disk friction system or a disc brake friction system, there is probably one or several specific elastic vibration modes in which relevant elastic vibration occurs in the plane defined by both friction and normal directions. Fig. 1 shows a friction system. It is comprised of two parts. Each part may be regarded as a subsystem. A finite element modal analysis of the friction system shows that there are several elastic vibration modes in which elastic vibration occurs in the plane defined by both normal and friction directions. Fig. 2 shows such a specific elastic vibration mode. From Fig. 2, it may be inferred that if one specific elastic

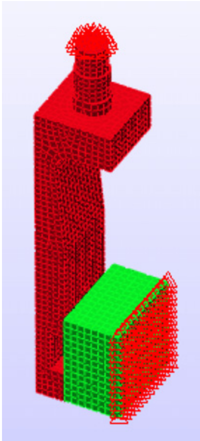

Fig. 1. Friction system.

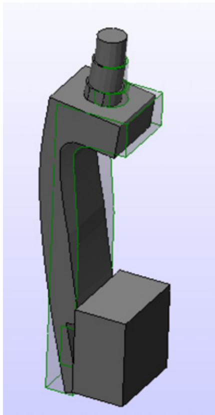

Fig. 2. Specific elastic vibration mode, natural frequency 1229.89 Hz.

vibration mode of the subsystem is excited, this mode vibration can be decomposed into two components along the normal and friction directions. The vibration component along the normal direction will bring about loading and unloading of the normal force. Consequently, the loading and unloading of the normal force will cause changes of the friction force. That result may be verified by a dynamic finite element simulation. Fig. 3 shows a dynamic finite element model of the system. Fig. 4 shows a dynamic simulation result. From Fig. 4b, it is found that the contact does not spread all over the whole nominal contact surface. Fig. 5 shows a comparison between the resultant of all dynamic normal forces at contact nodes and tangential displacement of a contact node. The dynamic normal force without steady component is extracted using zero phase filter technique [20]. From Fig. 5, it is seen that the normal force changing is in good agreement with the tangential displacement changing in phase.

The model shown in Fig. 2 may be reduced into a lumped mass model as shown in Fig. 6 [20]:

$$\ddot{x} + 2n\dot{x} + \omega_n^2 x = k_n x(t - \tau), \tag{1}$$

where n is a linear damping rate, ω_n is a circular frequency and k_n is an integrated coefficient which combines the contact stiffness and friction between two friction surfaces and the direct proportion between the tangential and normal displacements of the vibration. τ is a time delay between the varying normal and consequent friction forces. In Section 3.1, it is seen that the time delay τ is a key factor affecting instability of a friction system. However, it appears that seldom attention was given to the time delay in the literature. To authors' knowledge, the time delay designated in the present paper seems difficult to find in the literature. Hess

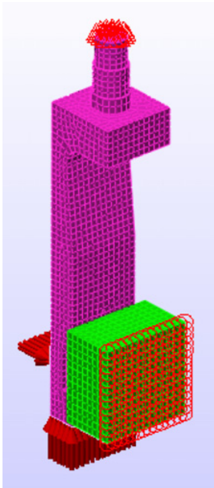

Fig. 3. Dynamic model, normal force 400 N, tangential exciting force $100\sin(2600\pi t)$ N and no friction at the sliding interface.

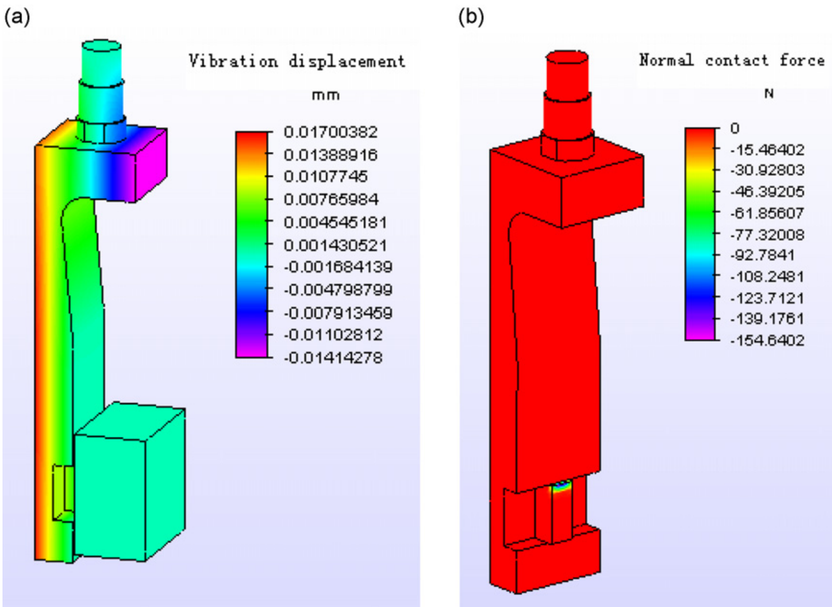

Fig. 4. Dynamic simulation results: (a) tangential vibration displacement of nodes and (b) normal contact force of nodes.

et al. found the time delay between the sliding speed and friction through an experimental test [23]. But Hess' result cannot directly result in the time delay between varying normal force and consequent friction force. The present authors carried out a series of tests to verify the existence of the time delay [20]. Although the test result partly supports the hypothesis of the time delay, the authors still think that more work needs to be done to verify and understand the concept of the time delay. A further experimental test of the time delay is in progress.

The above-mentioned self-excited vibration model with time delay is very similar to the model coupling theory. Their freedom coupling relations are the prominent similarity. Tworzydlo clearly concluded that coupling between rotational and normal modes was the primary mechanism of friction-induced vibration [24]. However, some differences between the mode coupling model and the present model with time delay are easily found. The mode coupling (means that two different vibration frequencies of a friction system become

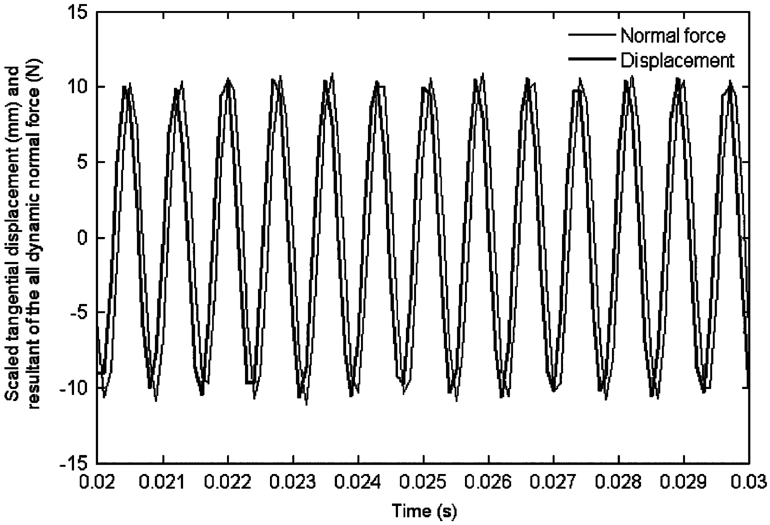

Fig. 5. Comparison between the scaled tangential displacement and resultant of all dynamic normal forces at contact nodes, the scaled tangential displacement equal to tangential displacement multiplied by 7000.

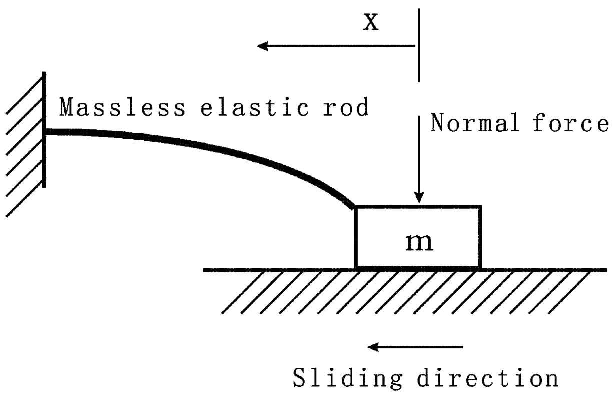

Fig. 6. Lumped mass model.

identical) is dependent on the friction coefficient in the model coupling theory. In the model with the time delay of friction, instability depends on the time delay while the normal vibration and tangential vibration are always dynamically coupled because these two vibration components are from the same elastic vibration. Now we know that the complex eigenvalue analysis, Massi's finite element transient analysis and the present model with time delay all can simulate the onset and bounding of squeal. However, damping is not taken into account in the complex eigenvalue analysis and Massi's finite element transient analysis. Therefore, the similarity between the mode coupling theory and the present model with time delay needs to be pursued. In the authors' work [20], the time delay between the varying normal and consequent friction forces was considered to be due to damping in the normal direction between two sliding bodies. If the normal damping between two sliding bodies is included in the squealing vibration model, instability of the system can be expected? That issue also needs to be further studied.

2.2. Stability analysis approach of the self-excited vibration model

Eq. (1) is a typical linear time invariant systems with time delay. Its vibration behaviour is decided by the roots of the characteristic equation. Generally, it is very difficult to obtain an analytical solution of the characteristic equation. The complexity arises due to the exponential-type transcendental terms in the characteristic equation. The transcendentality brings infinitely many characteristic roots. Instead, the roots of the characteristic equation are gained by numerical calculation. In the literature, number of methodologies have been suggested with limited ability to assess the stability in the parametric domain of time delay. Among these methods, the direct method proposed by Olgac and Sipahi [22] an excellent one. It is an exact, structured and robust method for analysing the stability of the linear time invariant systems with time delay. It not only reveals the stability regions in the domain of time delay, but also declares the number of unstable characteristic roots at any given region. In the present paper, the Olgac's direct method is adopted to analyse the stability regions of Eq. (1). The implementing procedure is given as follows:

(1) Establish the characteristic equation of Eq. (1).
(2) Substitute the exponential type transcendental term in the characteristic equation with $(1-Ts)/(1+Ts)$ to obtain a modified characteristic equation, where T is a real number, s is a characteristic variable. After substituting, the modified characteristic equation becomes an algebraic equation and is easily solved.
(3) Assume $s=\omega\mathrm{i}$ and substitute it into the modified characteristic equation (ω is a real number and i is the imaginary unit). A series of real number pairs T and ω may be obtained through solving the modified characteristic equation.
(4) According to the mapping condition of $\tau_l = 2/\omega[\tan^{-1}(\omega T) + l\pi]$to calculate time delay τ for all real number pairs T and ω, where $l = 0, 1, 2, \ldots$.
(5) Obtain the root tendency
$$RT = \mathrm{sgn}\left[\mathrm{Re}\left(\mathrm{d}s/\mathrm{d}\tau|_{\substack{s=\omega i\\ \tau=\tau_l}}\right)\right]$$
where $\mathrm{d}s/\mathrm{d}\tau$ is obtained based on the original characteristic equation.
(6) Form a table of τ_k, $k = 1, 2, 3, \ldots$, and $RT|_{\substack{s=\omega i\\ \tau=\tau_k}}$ in the ascending order of τ_l. Go to the smallest $\tau_k > 0$, assess the number of unstable roots as $\tau = \tau_k + \varepsilon$, $0 < \varepsilon \ll 1$, using the $RT|_k$. If $RT = +1$, the number of unstable roots increases by 2 and if $RT = -1$, it decreases by 2. Repeat the previous step for the next τ_k. Continue completing the analysis until the target value of τ_k is reached. Identify those regions in τ where the number of unstable roots = 0 as stable and, others as unstable.

3. Results of stability analysis

3.1. Outline of stability regions of the model with time delay

If there is not exponential-type transcendental term in Eq. (1), the system is stable. If there is an exponential-type transcendental term, however, the system may be stable or unstable depending on parameters of the system and time delay. There are some difficulties in obtaining the contact stiffness between two sliding surfaces exactly. It is dependent on the material elastic modulus and sizes of the contact bodies. The contact stiffness is found to be in the order of 10^6–10^8 N/m in the literature [12,25–26]. The coefficient of friction is generally between 0 and 0.6. The ratio of the vibration component in the normal direction over the vibration one in the friction direction for the specific elastic mode is strongly dependent on the friction system construction. Therefore, it is difficult to exactly obtain the value of integrated coefficient k_n. But it may be assessed to be about in the order of 10^6–10^8 N/m. As an example, assuming $\omega_n = 2\times\pi\times 1500$ rad/s, $n = 0.005\omega_n$, $k_n = 3\times 10^7$ N/m, the characteristic equation of Eq. (1) may be rewritten as follows:

$$s^2 + 2ns + \omega_n^2 = k_n \mathrm{e}^{-\tau s}. \tag{2}$$

Substitute $\mathrm{e}^{-\tau s}$ in Eq. (2) with $(1-Ts)/(1+Ts)$ and obtain the following algebraic equation:

$$s^2 + 2ns + \omega_n^2 = k_n \frac{1-Ts}{1+Ts} \tag{3}$$

when $s=\omega\mathrm{i}$, two solutions of T and ω obtained are $\omega_1 = 10899.94704$, $T_1 = -0.00535678$ and $\omega_2 = 7670.40749$, $T_2 = -1.57102\mathrm{e}-6$.

Based on Eq. (2), the derivative of s with respect to τ is obtained:

$$\frac{\mathrm{d}s}{\mathrm{d}\tau} = \frac{-sk_n \mathrm{e}^{-\tau s}}{2s + 2n + k_n \tau \mathrm{e}^{-\tau s}}. \tag{4}$$

According to the implementation procedure, the stability regions of Eq. (1) can be decided as shown in Table 1.

From Table 1, it is seen that when $0 \leqslant \tau < 0.000291$ s and $0.00081 < \tau < 0.00086$ s, the system is stable because the number of unstable roots is equal to zeros. When $0.000291 \leqslant \tau < 0.00081$ s, the system is unstable because there are two unstable roots. From Table 1, it is also found that the frequency of unstable vibration is different from the natural frequency of the friction subsystem and the unstable vibration frequency changes with time delay.

3.2. Influence of the natural frequency of subsystem on stability

Generally, the time delay between the normal and friction forces is very small. Therefore, the minimum time delay needed to cause unstable vibration shown in Table 1 is increased, the stability of friction system will be improved. There are many factors affecting the stability of Eq. (1). Fig. 7 shows the effect of the natural frequency of the specific elastic vibration mode on the minimum time delay. From Fig. 7, it is seen that the minimum time delay decreases with increasing nature frequency. That suggests that the higher the natural frequency, the more easy the system stability loss. Fig. 8 shows the variation of unstable vibration frequency with natural frequency. From the figure, it may be found that the unstable vibration frequency is different from the natural frequency. The higher the natural frequency, the smaller the difference between the unstable vibration frequency and the natural frequency. The difference may be attributed to the time delay.

The finite element complex eigenvalue analysis of a brake system shows that a higher frequency of mode coupling corresponds to a smaller coefficient of friction [27]. That is, the higher the frequency of mode coupling, the more easy squeal occuring. That finite element complex eigenvalue analysis result is consistent with the present result.

3.3. Effect of damping on stability

Fig. 9 demonstrates the variation of the minimum time delay with damping. From Fig. 9, it is seen that the minimum time delay increases with increasing damping. That suggests that the smaller the damping, the more easy the loss of system stability.

Table 1
Stability regions

τ (s)	RT	Number of stable or unstable roots	Frequency of unstable vibration ω (Hz)	T
		0		
0.000291	+1		1734.78	−0.0053
		2		
0.00081	−1		1220.78	−1.57e-6
		0		
0.00086	+1		1734.78	−0.0053
		2		
0.00144	+1		1734.78	−0.0053
		4		
0.00163	−1		1220.78	−1.57e-6
		2		
0.0020	+1		1734.78	−0.0053
		4		
0.00245	−1		1220.78	−1.57e-6
		2		
0.00327	−1		1220.78	−1.57e-6
		0		
⋮	⋮	⋮	⋮	⋮

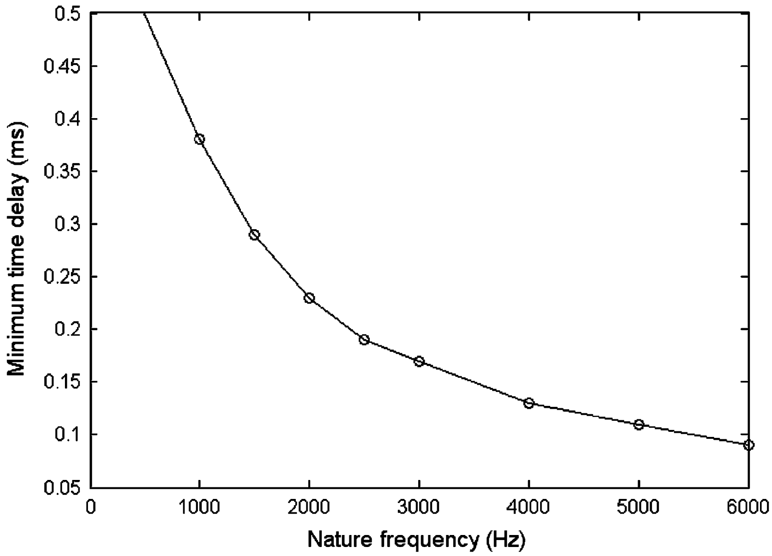

Fig. 7. Variation of the minimum time delay with nature frequency.

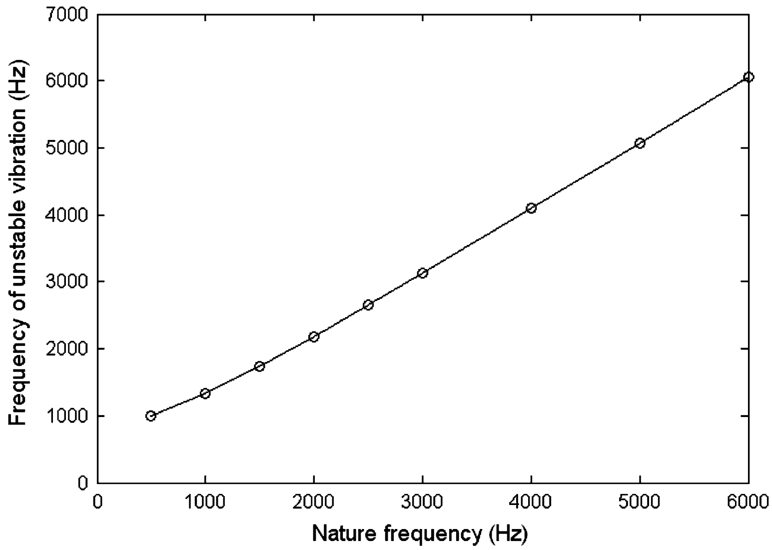

Fig. 8. Variation of the unstable vibration frequency with nature frequency.

It needs to be noted that the present result is different from several investigators' conclusion and far away from a real brake. Previous investigations showed that influence of damping on instability of friction system to be complicated. Iwan concluded that the addition of load damping generally causes the disk systems to become unstable for all speeds greater than the lowest critical speed [15]. Earles' study showed that high pin support torsional damping reduces the unstable region and disc damping had little effect on the size of the unstable zone [28]. In Earles' another study, it was concluded that increasing disc damping reduced the tendency for squeal noise to be generated and the effect of increasing pin damping depends on the physical parameters of the system, with tendencies to both increase and decrease squeal noise generation [29]. The authors think that the poor result may be attributed to the used simplified model. In that case, more complicated and practiced model such as finite element model is needed to obtain good results which are consistent with a real brake.

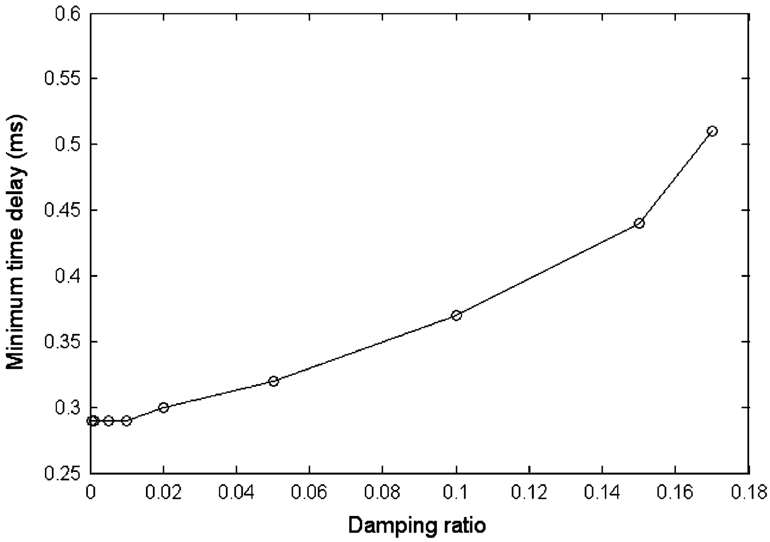

Fig. 9. Variation of the minimum time delay with damping.

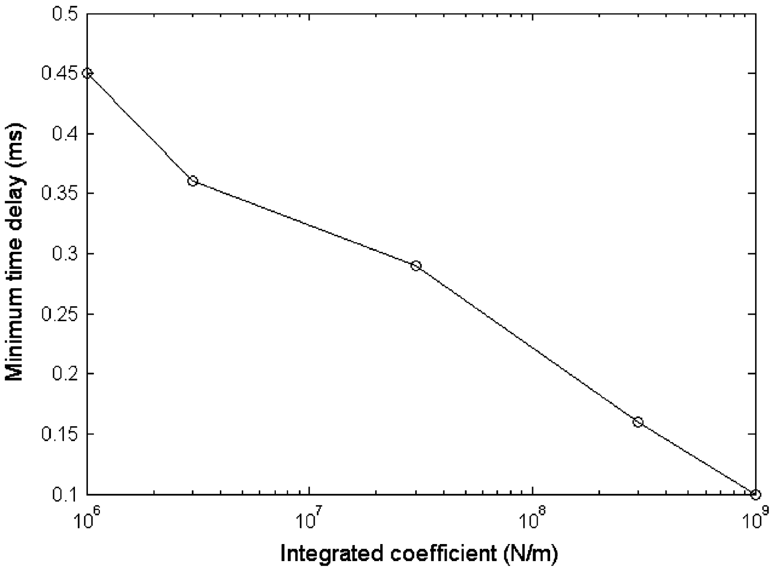

Fig. 10. Variation of the minimum time delay with the integrated coefficient.

3.4. *Influence of the integrated coefficient on stability*

The integrated coefficient of contact and friction combines the contact spring stiffness, friction coefficient, effective mass and ratio of the tangential vibration over the normal vibration of the specific elastic mode. Fig. 10 shows the variation of the minimum time delay with integrated coefficient. It is seen that the minimum time delay decreases with increasing integrated coefficient. That suggests that the higher the integrated coefficient, the more easy the loss of system stability. From Eq. (1), it may be found that the integrated coefficient is a gain of feedback loop. The higher the gain, the more the feedback energy. Therefore, the system more easy loses stability.

Especially, the integrated coefficient is directly proportional to the coefficient of friction. That is, the larger the coefficient of friction, the larger the integrated coefficient. From Fig. 10, it may be inferred that the larger the coefficient of friction, the more easy the occurrence of squeal. The result is consistent with those from finite element complex eigenvalue analyses [8–11].

4. Numerical simulation of the squealing vibration model with time delay

4.1. Linear simulation of the squealing vibration model with time delay

In Section 3.1, the stability regions of Eq. (1) are determined exactly. Based on the results shown in Table 1, one can identify whether the system is stable or not for a given time delay τ. However, the stability analysis only provides some messages on the boundary conditions of both stable and unstable regions. It cannot provide other messages such as the frequency of unstable vibration when the time delay is located at a point between two boundary conditions. In this case, the numerical simulation will remedy such a deficiency. In the present paper, Simulink package of MATLAB is applied to implement numerical simulations. The Simulink simulation model of Eq. (1) is shown in Fig. 11. In this model, blocks of steps 1 and 2 are used to create step signals, which are added or subtracted by block sum1 to produce an impulse $\delta(t)$. Blocks sum1 and sum2 are two adders, which are applied to add or subtract inputs. Blocks integrator 1 and integrator 2 are two integrators for continuous-time integration of the input signal. Blocks gains 1–3 are three amplifiers whose output is elementwise gain of input. Block transport delay is an operator, which is used to apply a specified delay to the input signal. Blocks scopes 1–3 are three scopes, which are used to demonstrate the results. In this paper, some numerical trials are carried out using different solvers provided by Simulink and different integration time steps. The results show that the simulation results are similar whichever of fixed-step solvers ode4, ode3 and ode2 is used as well as whichever of time steps of 0.0001–0.00001 s is applied. In this paper, solver ode4 and integration time step of 0.00001 s are applied to perform the related simulations.

When $\tau = 0.000292$ s, the simulation result is shown in Fig. 12. It is seen that in this case the vibration becomes larger and larger with time, i.e., the system is unstable. The power spectrum density analysis of the

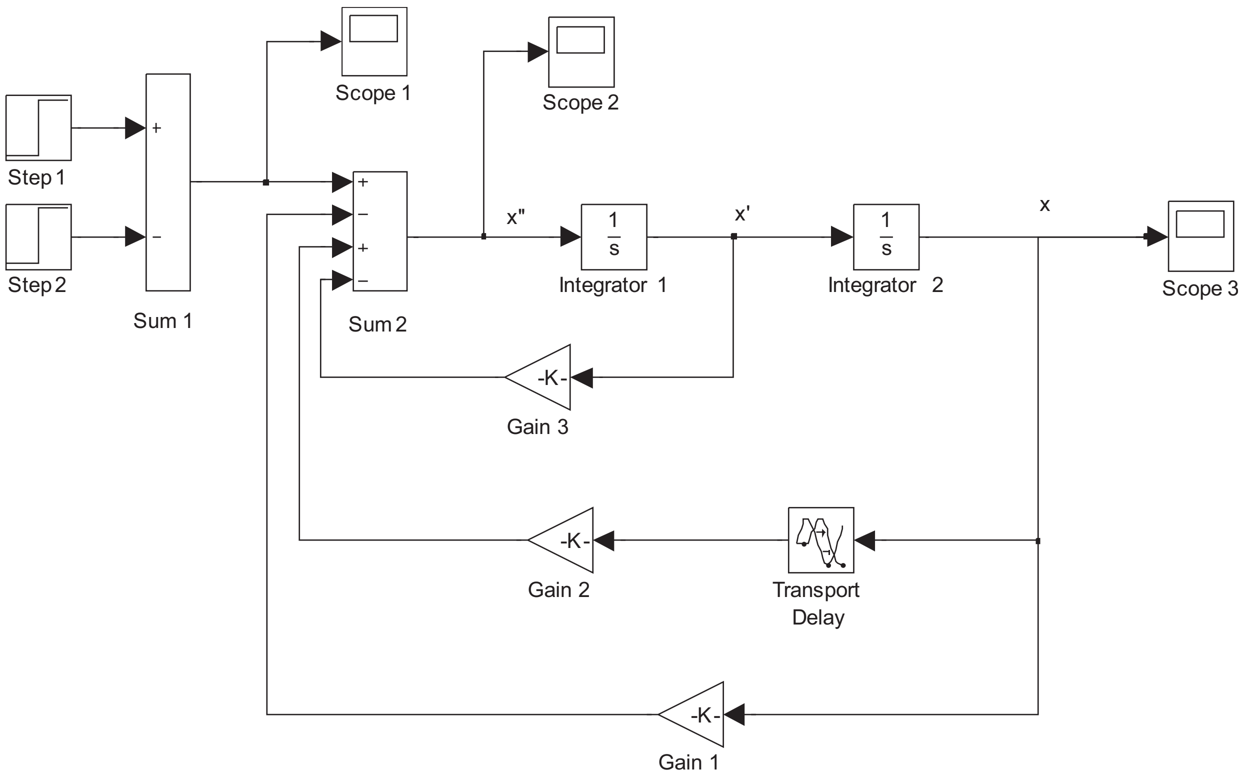

Fig. 11. Simulink model of Eq. (1).

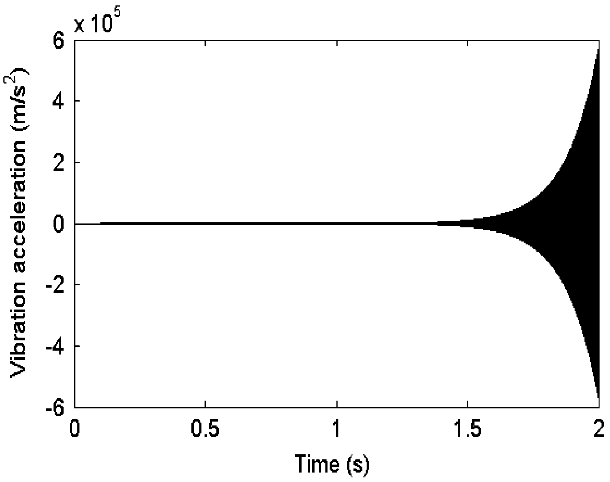

Fig. 12. Unstable vibration, time delay $\tau = 0.000292$ s.

unstable vibration shows that the unstable vibration frequency is about 1733.4 Hz, which is very close to 1734.78 Hz obtained by the direct method analysis of Eq. (1). When $\tau = 0.000290$ and 0.00082 s, the simulation result is shown in Fig. 13. It is seen that in these cases the vibrations decrease little by little with time, i.e., the system is stable. The results are consistent with those shown in Table 1. Fig. 14 shows the variation of the unstable vibration frequency with time delay. It is found that when the time delay varies from 0.000292 to 0.00081 s, the frequency of unstable vibration changes approximately linearly.

4.2. Nonlinear simulation of the squealing vibration model with time delay

4.2.1. Simulation of vibration bounded

Fig. 12 shows that the vibration becomes larger and larger with time in the unstable region. In fact, the squealing vibration measured in the field test is always found to be bounded. In the literature, that bounded vibration is attributed to nonlinearity of friction system. However, it remains partially unknown what the nonlinearity is and further how it acts on the system is nuclear. Here, the nonlinearity is studied. It is well known that the normal component of squealing vibration is always coupled with the tangential component. The normal vibration component results in loading and unloading of the normal force. When the normal force is unloaded to zero, the friction force is equal to zero. In this case, the dynamic normal force due to the normal vibration is equal to the steady-state preloaded normal force. That event is a nonlinearity arising in squealing vibration. Having taken into account the nonlinearity, Simulink model of the squealing model is shown in Fig. 15. In Fig. 15, block MinMax is an operator which outputs min or max of input. For a single input, operators are applied across the input vector. For multiple inputs, operators are applied across the inputs. Fig. 16 shows a simulation result. Comparing Fig. 16a with b, it is clearly seen that when the normal force is unloaded to zero, the system vibration is bounded. In the field test of squeal, it is found that sometimes the sound pressure level of squeal increases with increase in normal force. That result may be simulated with the model shown in Fig. 15. Fig. 17 shows the variation of the maximum amplitude of nonlinear vibration with normal force. It is seen that with increasing normal force, the maximum amplitude of nonlinear vibration is increased. Since the sound pressure level of squeal is directly proportional to the amplitude of vibration, it may be concluded that the sound pressure level of squeal will increase with increasing normal force.

Dweib et al. applied the method of triple-input describing functions to study the limit cycle characteristics of friction-induced vibration. In their work, the amplitudes of the normal and angular displacements, and fundamental frequency of limit cycle vibration were determined [30]. Most recently, Massi et al. conducted a good finite element transient simulation of the onset and bounding of squeal vibration [12]. In Massi's paper, two cases of the bounding of squeal vibration were presented. One case of bounding of squeal vibration was found not to depend on the contact separation (that is, the net normal force is not equal to zero). While

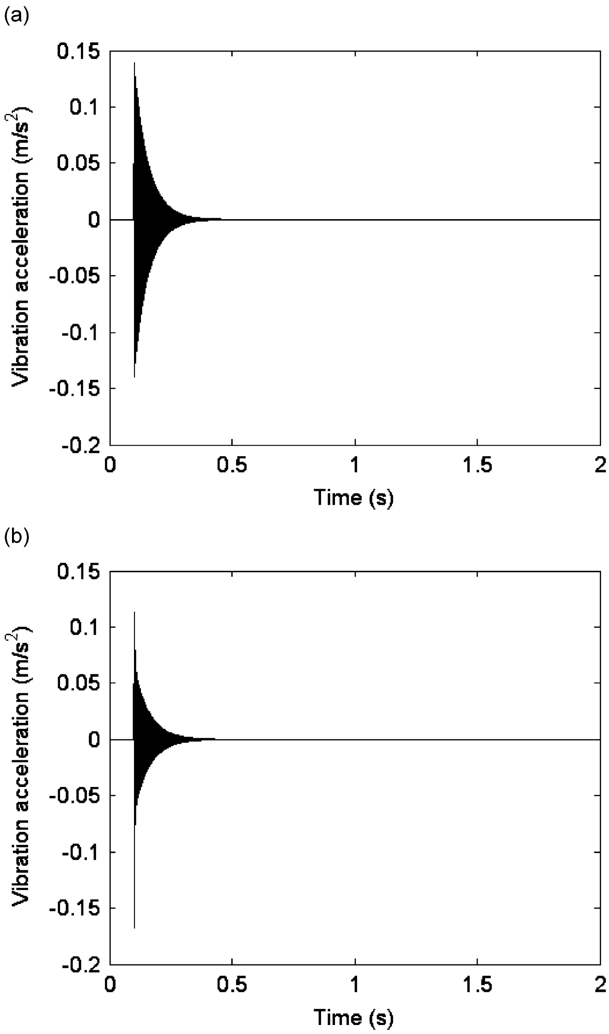

Fig. 13. Stable vibration: (a) time delay $\tau = 0.000290\,\mathrm{s}$ and (b) time delay $\tau = 0.00082\,\mathrm{s}$.

another case of the bounding of squeal vibration was found to be dependent on the contact loss (i.e., the net normal force is equal to zero). The present work cannot achieve the result of the first case. To achieve the result, a finite element transient model similar to the one shown in Fig. 1 is needed. However, some difficulties, e.g., how to include the time delay of friction at the contact interface in the finite element model, need to be solved. That work is what the authors are planning to do in the coming time.

4.2.2. Prediction of squealing vibration disappearance

Experimental tests showed that sometimes squeal vibration did not vanish once started and reached its limit cycle [31] and sometimes it only occurred at a local zone in a cycle [8,32,33]. The transient analysis in Section 4.2.1 and Massi's finite element transient analysis all can simulate the former case. But the simulation of the latter case needs to be pursued.

In general, the complex eigenvalue analysis of friction systems deals only with the onset of squealing vibration. It is helpful for understanding the squeal generation mechanism to study the disappearance of squealing vibration. In Section 3, it is found that the time delay between the varying normal force and its causing changing friction has a distinct influence on squeal formation. Friction is a very intricate phenomenon. There are many factors affecting friction and its evolution. It is well known that friction changes

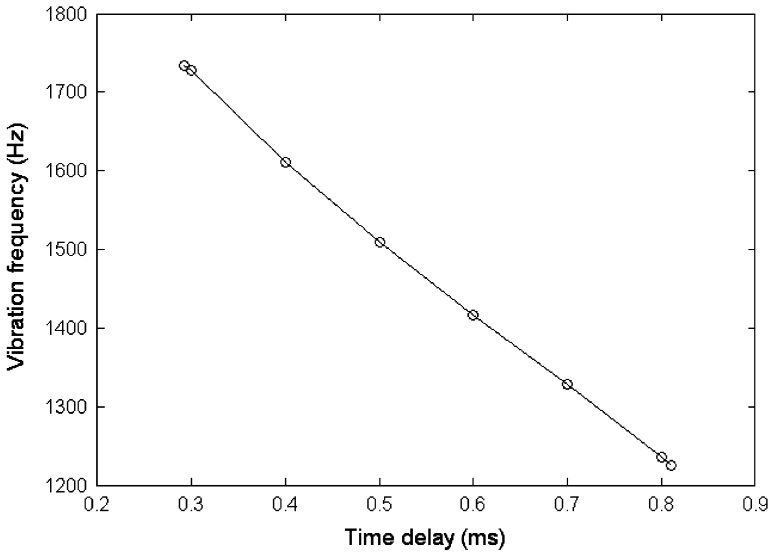

Fig. 14. Variation of the vibration frequency with time delay.

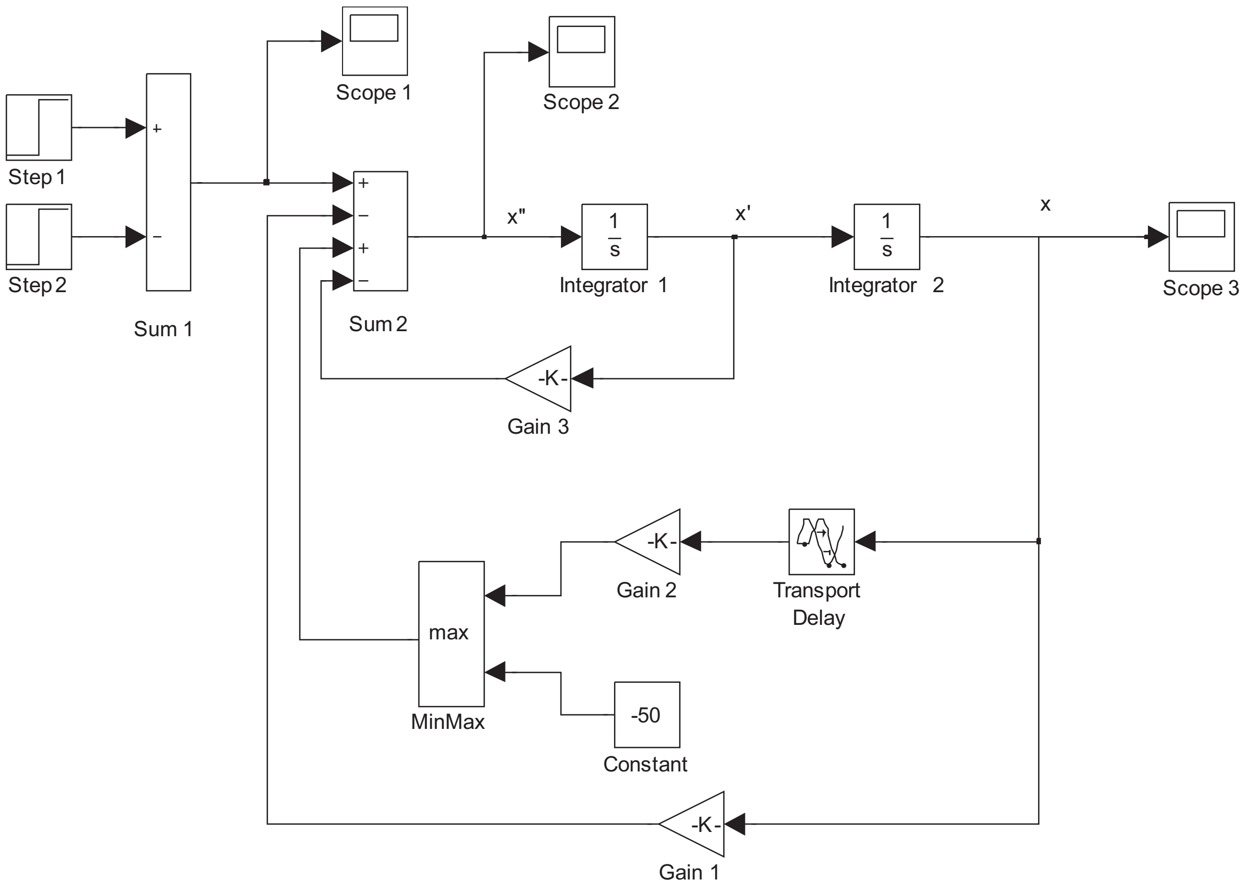

Fig. 15. Nonlinear Simulink model.

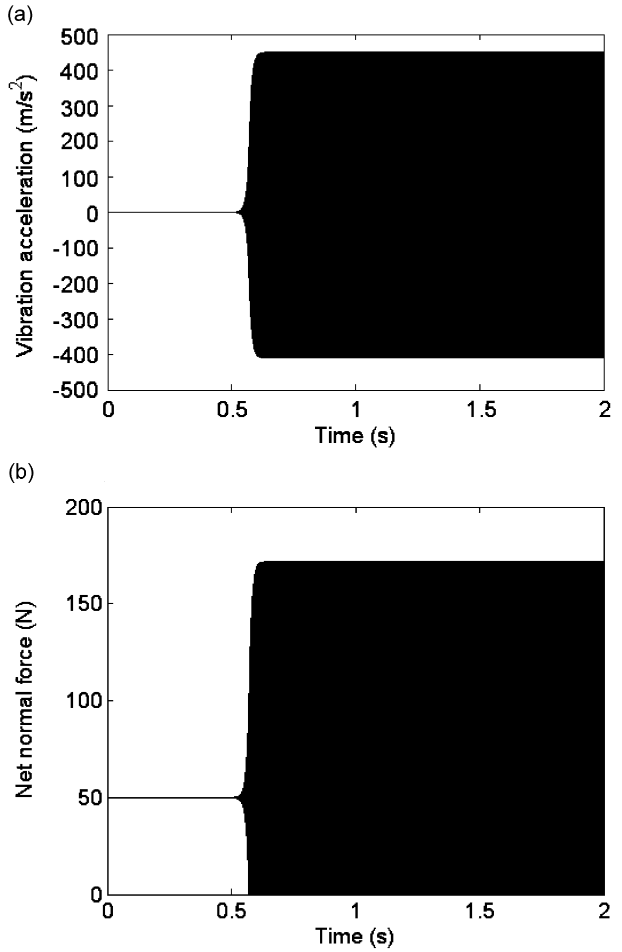

Fig. 16. Nonlinear simulation results, steady-state normal force 50 N: (a) time history of nonlinear vibration and (b) time history of the net normal force.

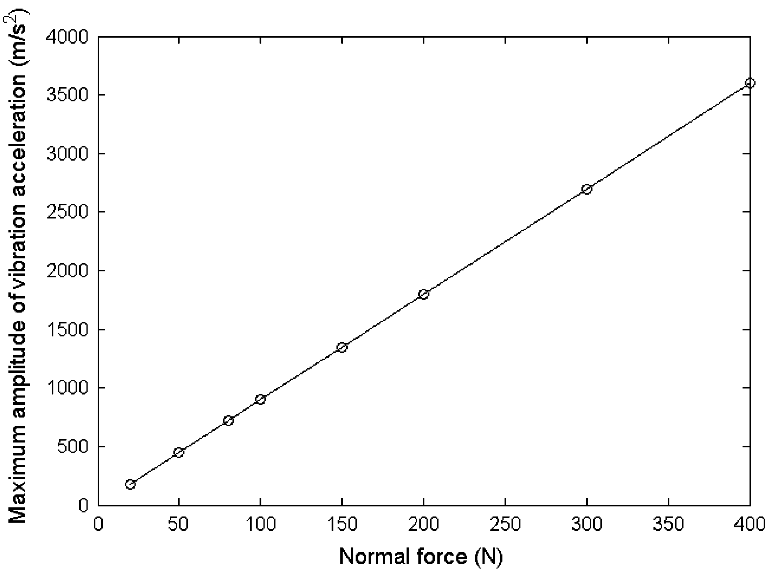

Fig. 17. Variation of the maximum amplitude of nonlinear vibration with normal force.

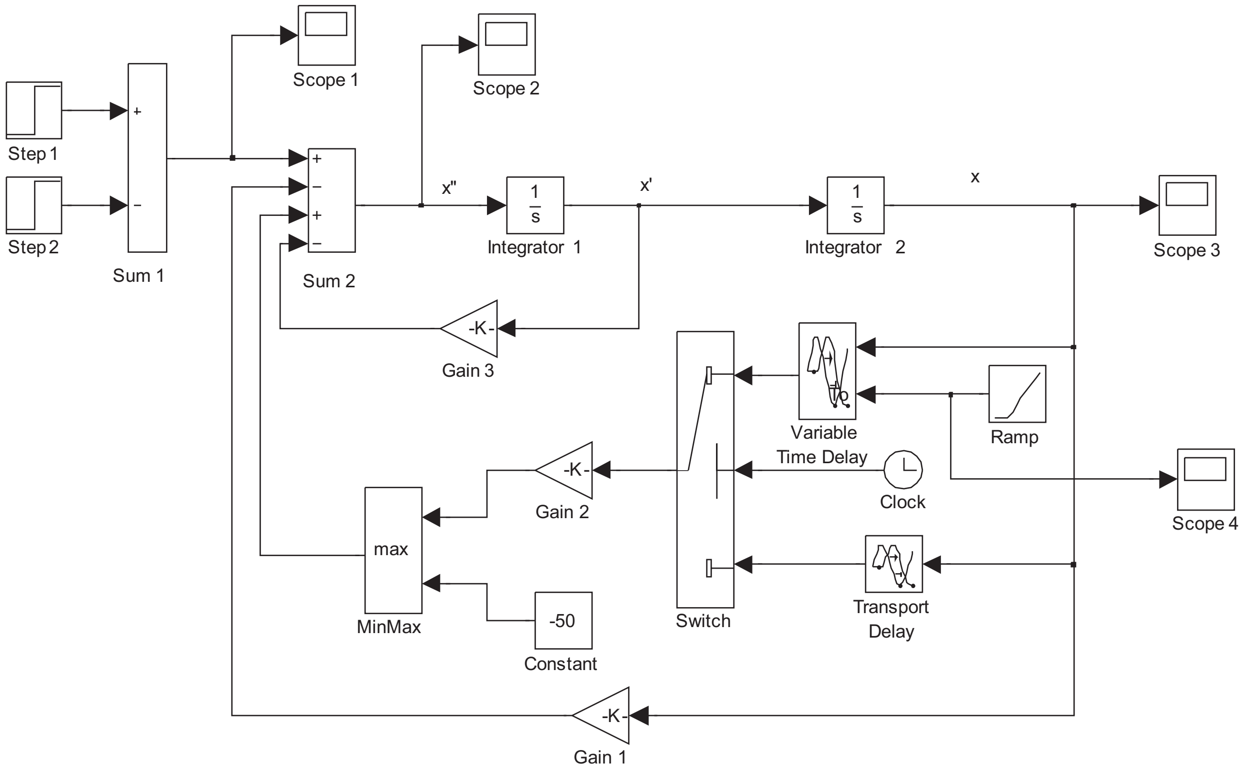

Fig. 18. Nonlinear Simulink model used to predict squealing vibration disappearance.

with sliding distance. Therefore, it may be inferred that the time delay also varies with time. There are two possibilities of time delay changing, i.e., the time delay decreases with time or vice versa. Taking time delay changing into account, the Simulink model is shown in Fig. 18. Block clock is a digital clock, which is used to output a current simulation time. Block switch is a switch, whose function is passing through input 1 when input 2 satisfies the selected criterion; otherwise, passing through input 3. It is used to set the time when the time delay starts to change. Block variable time delay is an operator, which is used to apply a delay to the first input signal. The second input of the block specifies the delay time T_0. The block implements the function $y = u(t - T_0(t))$. Block ramp is a signal generator, which outputs a ramp signal starting at the specified time. When the time delay decreases with time, a simulation of squealing vibration disappearance prediction is performed based on the following conditions: the model is excited at the time of 0.05 s. From time 0 to 0.2 s, the time delay is set to 0.0003 s. At the time of 0.2 s, the time delay is designated to change as shown in Fig. 19. Fig. 20 shows the simulation result. From Fig. 20, it is seen that if the time delay is decreased below 0.000291 s, the squealing vibration will vanish little by little.

5. Qualitative comparisons between the simulation and test results

5.1. Comparisons in time domain

More recently, several transit analyses related to squeal were published. But in these articles, there are very limited comparisons between the simulation results and test results, which mainly focus on those in time domain. The present authors think that one model would successfully simulate main characteristics of squealing vibration behaviour if it can capture true mechanism of squealing vibration formation. Generally, the characteristics of squealing vibration behaviour include those in time domain and those in frequency domain. The former is well known for the squeal research community but the latter is relatively strange. In the

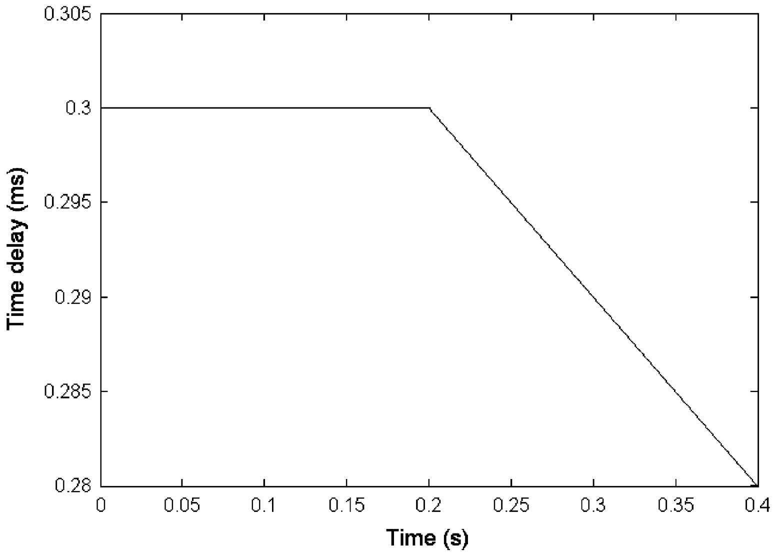

Fig. 19. Assumed variation of the time delay with time.

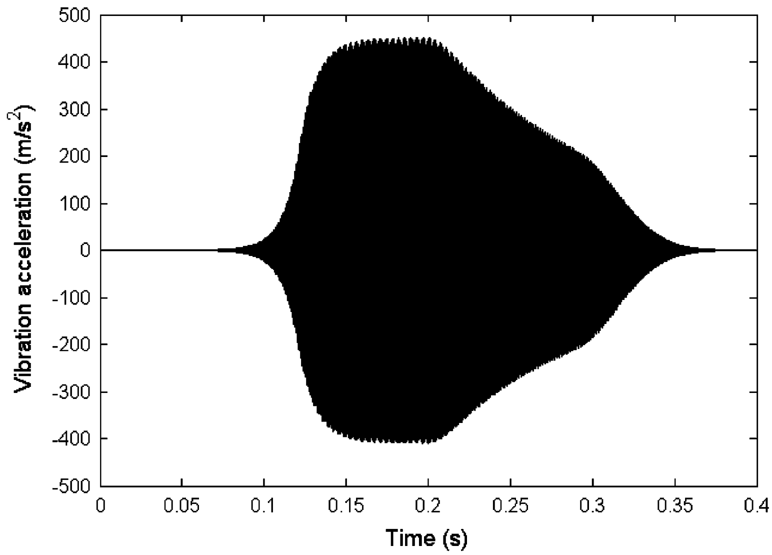

Fig. 20. Variation of the vibration acceleration with time.

authors' previous work [19], they found that the time frequency representation of squealing vibration contained nonlinear vibration messages. It was found the there is always a sudden decrease in vibration frequency at the place where the vibration is bounded. In terms of time frequency characteristics, the change in frequency suggests that nonlinearity arises. Here, the simulation results will be compared with test results in both time and frequency domains. Fig. 21 shows a time history of tangential vibration measured in a reciprocating sliding squeal test. The test conditions and more test results may be found in Refs. [19,32]. It needs to be noted that in Fig. 21 the positive amplitude of vibration is different from the negative one. This characteristic is also found in pin-on-disc test results [33]. It may be a common characteristic of squealing vibration. Comparing the simulation result shown in Fig. 20 with the test result shown in Fig. 21, it is seen that the simulation result also has the same characteristic as the test result.

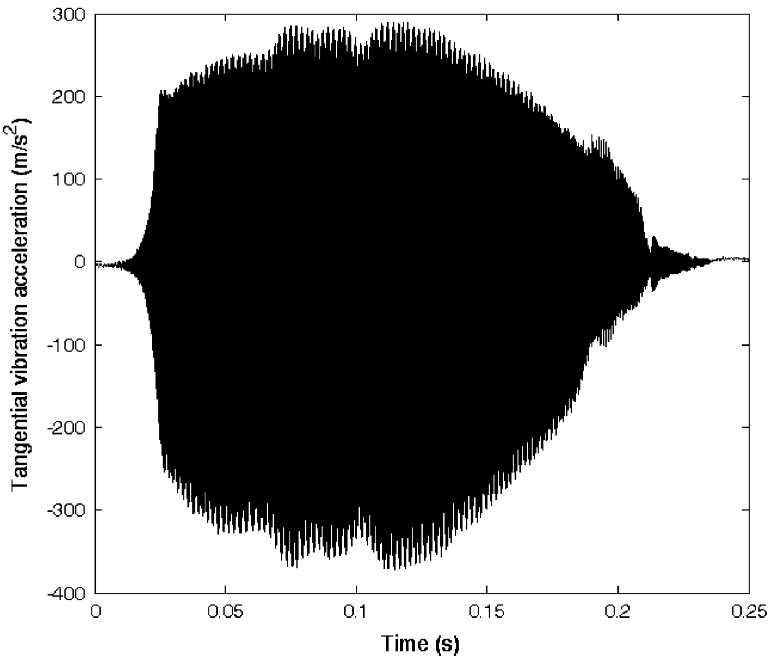

Fig. 21. Time history of measured squealing vibration. The test conditions: frequency of reciprocating sliding 0.5 Hz, displacement amplitude of reciprocating sliding 2 mm, normal force 200 N.

5.2. Comparisons in frequency domain

Figs. 22 and 23 show the time–frequency representations corresponding to the simulation result in Fig. 20 and the test result in Fig. 21, respectively. From Fig. 22b, it is seen that before the vibration is bounded the vibration frequency is kept constant. At the place where the vibration is bounded, the vibration frequency is suddenly decreased. After that, the vibration frequency is kept constant again. At the vicinity of vibration end, the vibration frequency is increased to the initial frequency value little by little. Comparing Fig. 22b with Fig. 23b, it is found that the time–frequency representation of the simulation result is very similar to that of the test result. That similarity of time–frequency representation is very important for the simulation model. Since the time–frequency characteristic can give an overall view of the vibration behaviour [34], the present authors think that it would be counted as an important comparison item. In Ref. [19], it remained unknown what causes nonlinearity, i.e., a sudden decrease in vibration frequency at the location where the vibration is bounded. Here, that nonlinearity is known to be due to contact separation between two sliding surfaces.

Based on the similarity in both time domain and frequency domain between the simulation results and test results, the mechanisms of squealing vibration generation, growth and disappearance may be concluded to be the followings: A friction system with the specific elastic mode may be excited by small impulse interference when the time delay is larger than a certain value. The self-excited vibration will increase until the contact between two sliding surfaces is separated. When the contact separation is encountered, the vibration is bounded. The time delay may change with time due to wear and presence of debris. When the time delay is decreased to a certain value, the self-excited vibration will be attenuated little by little until the vibration vanishes.

6. Conclusions

Although the present model with time delay is very simple, it illustrates a possible mechanism for squeal generation which is partially different from the mode-coupling mechanism. In the initial development of the mode-coupling theory, one obtained the knowledge of the mode-coupling from a simple model with two

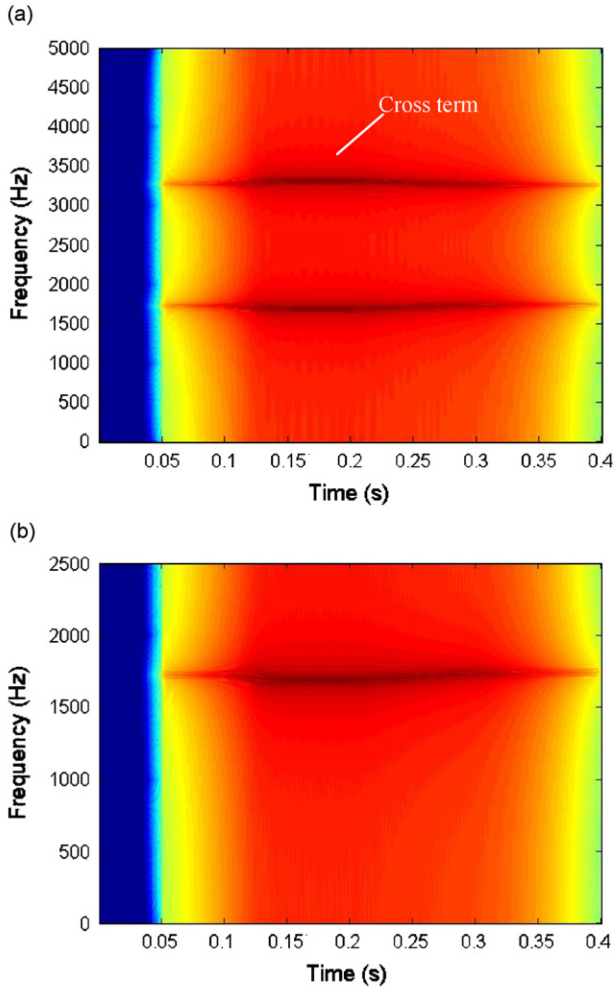

Fig. 22. Time–frequency representation of the vibration shown in Fig. 20: (a) time–frequency representation and (b) details of the time–frequency presentation.

degrees of freedom [2]. Then he extended it to complicated models with multiple degrees of freedom and finite element models. The authors consider that the present simple model may be also extended to complicated finite element model. By that time, the similarity between the mode-coupling theory and the present model with time delay will be seen clearly. The present model will help us to further understand the significance of mode-coupling and extend insight on the factors contributing to break squeal anyway.

In the present paper, the stability of a squealing vibration model with time delay was analysed. Linear and nonlinear simulations of the model were carried out. Qualitative comparisons in both time and frequency domains between the simulation results and test results were performed. The following conclusions were obtained.

1. The time delay between the varying normal force and its causing varying friction has a distinct influence on squeal occurrence. With an increase of the time delay, the stability region without squeal and unstable region with squeal arise alternately. In general, there is always an unstable region with time delay.
2. The larger the natural frequency of the specific elastic vibration mode, the more easy the instability of friction system.
3. The larger the integrated coefficient, the more easy the instability of the friction system.

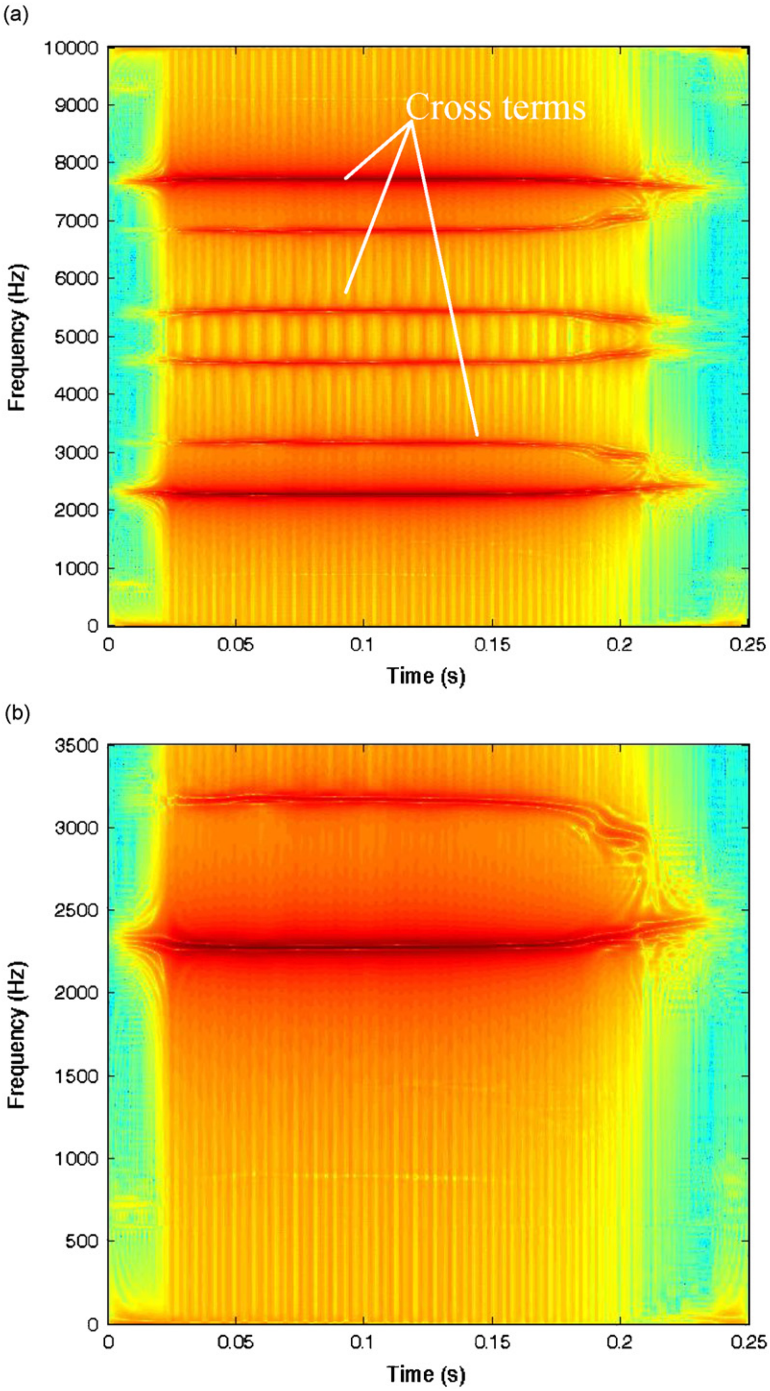

Fig. 23. Time–frequency representation of the vibration shown in Fig. 21: (a) time–frequency representation and (b) details of the time–frequency presentation.

4. The nonlinearity leading to bounded vibration in the process of squealing vibration propagation is known to be due to contact separation between two sliding surfaces.
5. The mechanism of squealing vibration generation, growth and disappearance is proposed to be the followings: A friction system with the specific elastic mode may be excited by small impulse interference when the time delay is larger than a certain value. The self-excited vibration will increase until the contact

between two sliding surfaces is separated. When the contact separation is encountered, the vibration is bounded. The time delay may change with time due to wear and presence of debris. When the time delay is decreased to a certain value, the self-excited vibration will be attenuated little by little until the vibration vanishes.

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